Physics

HP COMPUTER CURRICULUM

Waves

STUDENT LAB BOOK

HEWLETT PACKARD

Hewlett-Packard
Computer Curriculum Series

physics
STUDENT LAB BOOK

waves

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A, Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Physics Lab Book was developed to provide you the opportunity to use a computer as a problem solving tool. You will write computer programs which will enable you to investigate some very important ideas in physics. Using just one program, you will be able to perform many different experiments and hopefully make your own generalizations from the results. If you become very involved in investigating ideas in physics, the Lab Books will have achieved their aim.

To use the Lab Book for Waves you will need the following. First you should have a background in algebra and some trigonometry although the necessary trigonometry will be reviewed in this Lab Book. Secondly, the Lab Book assumes that you already know how to write simple computer programs using the BASIC language. If you do not, you will want to study BASIC before you attempt the material. Consult the BASIC Manual for the computer you use. Last, use of this Lab Book requires that you have access to a computer for at least two hours per week. If more time is available, you may be able to experiment further on your own, either to improve your program or to investigate other aspects of physics that interest you.

As you will discover, there is no one "right" way to use a computer as a problem solving tool. There are many different ways to solve one problem by programming. Experiment and learn as you go. You'll find you are learning something new each time, both about your subject matter and about using the computer to solve problems.

This book was designed to help you by providing several different kinds of material. First, there are the exercises with the preparatory explanatory material. These exercises are sequenced so that you can apply what you have learned in the previous problem in solving the next one. Often you can take your preceding program and simply add to it to create a program that will provide answers to the more general or more advanced problem.

Sections of advanced problems are provided for any student interested in further work in this area. You may wish to tackle these after you have completed the basic exercises.

When you begin using the Lab Book, you should develop the habit of planning a solution in the form of a flow chart before beginning to write the program. This is good programming practice. Drawing the flow chart provides a check of your logic, and the finished flow chart can then be used as a guide in writing each step of your program.

WAVES

Waves are fundamental in any study of physics. Though there are many types of waves, it turns out that they all share several fundamental characteristics. We will be concerned with these common characteristics in this unit. The computer is a particularly useful tool for studying them.

Most of the exercises in this unit involve graphical results. If you are fortunate enough to have an x-y plotter or other graphic display device connected to your computer, by all means use it. Your instructor will give you instructions. Most of you, however, will be using the teletype as a crude plotter. The results are not too accurate, but are sufficient for our purposes. You will also learn how to draw quick sketches of waves. From time to time you will be asked to draw a sketch of the anticipated results of a problem before solving it on the computer. These sketching exercises are important because they will develop your ability to visualize results.

REVIEW OF TRIGONOMETRY

Waves are usually described in terms of trigonometric functions. We will begin with a brief review of the essential points of trigonometry which will be required for our investigation. At the same time, you will learn how to use the teletype as a crude plotting device. We will be concerned only with *circular trigonometric functions.* Circular functions are defined by a circle of unit radius, as shown in Figure 1. The radius of the circle equals 1 and is centered on the origin of an x-y coordinate system.

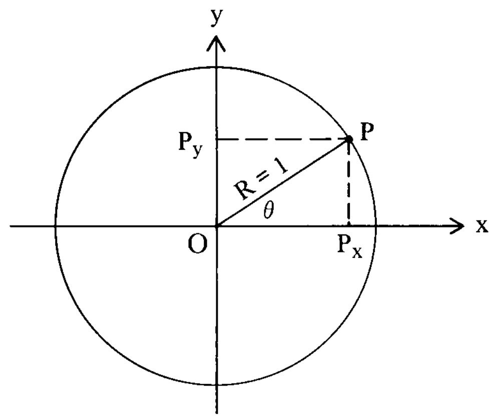


Figure 1 – A Unit Circle

If we locate a point P on the circle and connect it to the origin with a radius, an angle θ is defined which is measured between the positive x axis and the radius joining P to the origin. If P is moved from the intersection of the circle with the positive x axis counterclockwise around the circle the angle swept out is defined to be positive. If the point P is moved clockwise around the circle, a negative angle is swept out.

Before we can proceed we must define the angle in numerical terms. The definition of the angle is given by

$$\theta = \frac{S}{R} . \tag{1}$$

S is the arc length measured on the circumference of the circle and R is the radius of the circle. If P moves around the circle and returns to the starting point, the total arc length swept out is the circumference of the circle, which is $2\pi R$. Thus, by (1) the total angle swept out is 2π. The unit of angle measure defined by (1) is the *radian.* Since there are 360° in a complete circle, then $360^\circ = 2\pi$ radians, or 1 radian = 57.2957°. In BASIC we will use radian measure for angles, but occasionally you will want results in degrees. In that case, use the above conversion factors to convert your results from radians to degrees and vice versa.

In a unit circle where R = 1 (Figure 1), we can define the two basic trigonometric functions in terms of the projection of the radius of the circle upon the x and y axes. The projection of OP upon the x axis is simply the x coordinate of P. If the radius

of the circle is 1, then the x coordinate of P is defined as the *cosine* of θ and is abbreviated $\text{Cos}(\theta)$. When P is on the positive x axis, $\theta = 0$ and $\text{Cos}(0) = 1$. When P is on the positive y axis, $\theta = \frac{\pi}{2}$ and $\text{Cos}(\pi/2) = 0$. In a similar manner $\text{Cos}(\pi) = -1$, and $\text{Cos}(3\pi/2) = 0$. Finally, $\text{Cos}(2\pi) = \text{Cos}(0) = 1$. As you can see, the cosine of *any* angle must fall between +1 and −1 inclusively.

The projection of OP upon the y axis is the y coordinate of P. Again, if $R = 1$ the y coordinate of P is defined as the *sine* of θ and is abbreviated $\text{Sin}(\theta)$. Figure 1 shows that $\text{Sin}(0) = 0$, $\text{Sin}(\pi/2) = 1$, $\text{Sin}(\pi) = 0$, $\text{Sin}(3\pi/2) = -1$, and finally $\text{Sin}(2\pi) = \text{Sin}(0) = 0$. As with the cosine, the sine of *any* angle must fall between +1 and −1 inclusively.

The remaining circular trigonometric functions can be defined in terms of the sine and cosine functions, but they are not needed for our study of waves. The characteristics of waves can be described in terms of the sine and cosine functions.

GRAPHING THE SINE AND COSINE FUNCTIONS

The angle θ can be defined in terms of the angular velocity (ω) and time (t). As you might suspect, the relationship is given by

$$\theta = \omega t \,. \tag{2}$$

As t changes, the point on the unit circle moves around the circumference at a constant rate if ω is a constant. We need to know how Sin(ωt) and Cos(ωt) behave as t changes. Before we determine that, however, we should learn to use the teletype as a plotter.

The primary function of any plotter is to locate a point in two dimensions. As the printing head on the teletype moves across the page, we can generate movement in one dimension. As the paper feeds up through the teletype, we get movement in the second dimension. Thus, with some limitations, we can use the teletype as a crude but effective plotter.

As you know from your previous study of BASIC, the PRINT command controls vertical movement of the paper in the teletype. As soon as a quantity is encountered in a PRINT statement that is not followed by punctuation, or when the PRINT statement stands alone, there is an automatic line feed that moves the paper up one space. We will use this feature in our graphic program.

If you have studied typing you know that a typewriter has tabulator settings that can be used to move the carriage across to previously selected locations. The teletype has the same feature except that you cannot control it manually. The tabulator settings are made under program control using the TAB command.

There are 72 printing positions on a single line, numbered 1 through 72 sequentially. The form of the TAB command is TAB(number) or TAB(expression), for example TAB(38) or TAB(X – 2*Y). TAB(38) causes the teletype to space across to the 38th printing position. TAB(X – 2*Y) requires two steps: the expression is evaluated using current values of X and Y, then the teletype spaces across to the printing position specified by the resulting value. Note that the final argument of the TAB function must be a positive integer between 1 and 72 inclusively.

The TAB command therefore allows us to move the printing head laterally on the page under program control. The teletype has certain limitations when used as a plotter; namely, the printing head can only move from left to right, and the paper can only be moved up with an automatic line feed. However, even with these limitations we can draw very effective graphs.

The program listed in Figure 2 plots Sin(ωt) versus t with ω assumed to be 1. The program is organized so that Sin(ωt) = 0 corresponds to tab setting 30, Sin (ωt) = 1 corresponds to tab setting 45, and Sin(ωt) = –1 corresponds to tab setting 15. Therefore the *amplitude* of the sine function, which is 1, corresponds to 15 spaces on the tabulator. This value of 15 is the scale value of S set in line 120. The program is clear with the exception of line 190. Here we generate the argument of the TAB function.

The term S*FNA(T) generates a scaled value of our function which is then added to or subtracted from 30 as FNA(T) is + or – respectively. Finally, 0.5 is added so that when the integer part is taken, the result is rounded off to the nearest tab setting instead of being truncated to the next lower setting. This results in a somewhat smoother graph.

```
LIST
100  REM PLOT OF SIN(T)  VERSUS T
110  LET S=15
120  DEF FNA(T)=SIN(T)
130  FOR I=1  TO 60
140  PRINT TAB(I);"-";
150  NEXT I
160  PRINT
170  LET P=3.14159
180  FOR T=0  TO 2.01*P  STEP 2*P/40
190  LET Y=INT(S*FNA(T)+30.5)
200  IF Y>30  THEN 240
210  IF Y<30  THEN 260
220  PRINT TAB(30);"*"
230  GOTO 270
240  PRINT TAB(30);"I"; TAB(Y);"*"
250  GOTO 270
260  PRINT TAB(Y);"*"; TAB(30);"I"
270  NEXT T
999  END

READY
```

Figure 2 – Program for Sine Plot

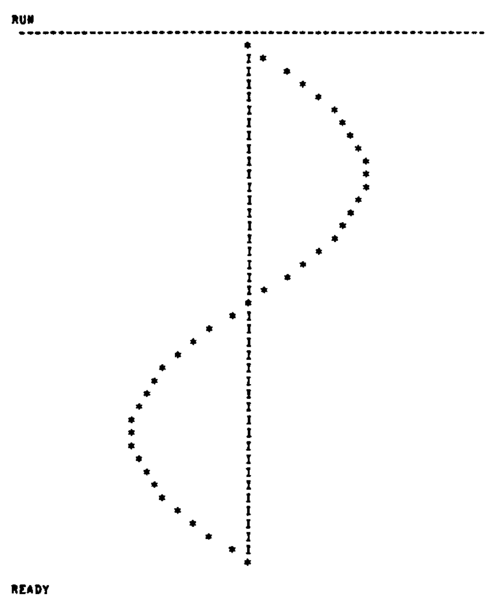


Figure 3 – Computer Plot of y = Sin(t)

The output of the program is contained in Figure 3. Study both the program and the graph until you can explain where in the program each feature of the graph was generated. The graph shows how the sine function behaves as the argument changes and is known as a *sine wave* or *sinusoid.*

Recall that we have been examining the function

$$y = \mathrm{Sin}(\omega t)\ . \tag{3}$$

As t increases, the point P moves around the unit circle in Figure 1 at a constant angular velocity. As soon as the point moves completely around the circle, the pattern in Figure 3 will start to repeat. A function which repeats in this way is known as a *periodic* function. We would like to know the *period* of the function which is defined as the time necessary to go through one complete cycle. In this case, if we designate the period by T, we can see that

$$\omega T = 2\pi\ , \tag{4}$$

so that the period is defined by

$$T = \frac{2\pi}{\omega}. \tag{5}$$

Thus if ω is 1 radian/second, it will require 2π seconds for the sine wave to go through one complete cycle. If T is π-seconds, then ω must be 2 radians/second, and so on.

We began this discussion by assuming that θ in Figure 1 was defined as a function of time, i.e., $\theta = \omega t$. Now we will examine the same function but will assume instead that θ is a function of distance, so that

$$\theta = kx\ , \tag{6}$$

where k is a constant and x is distance. If we assume k = 1 and substitute x for t in the program in Figure 2, then the output is unchanged and we will get the same graphical result as in Figure 3. In our first example we determined the *time* required to go through one complete cycle and the corresponding *period*; in this example we are interested in the *distance* required to go through one complete cycle and the corresponding *wavelength*. The wavelength is given the symbol λ, and when $x = \lambda$, the function has gone through one cycle. Consequently θ must be equal to 2π. This allows us to identify k in (6).

$$k\lambda = 2\pi\ , \tag{7}$$

or

$$k = \frac{2\pi}{\lambda}\ . \tag{8}$$

The constant k is given the same *wave number*.

EXERCISE 1 – A Graph of the Cosine Function

a. *Modify the program in Figure 2 to find the shape of $y = Cos(\omega t)$ with $\omega = 1$.*

b. *Sketch what you think the computer output will look like before you run the program (refer to Figure 1). Compare your sketch with the program output.*

c. *What is the relationship between $Sin(\omega t)$ and $Cos(\omega t)$? Can you express this relationship in the form $Cos(\omega t) = Sin(\omega t + ?)$?*

d. *Given $y_1 = Sin(\omega t)$ and $y_2 = Cos(\omega t)$, compare the amplitudes and periods of y_1 and y_2.*

EXERCISE 2 – A Change in the Period, $y = Sin(2t)$

a. *Modify the program in Figure 2 to ploy $y = Sin(2t)$.*

b. *Sketch the output you expect. Then run the program and compare with your sketch.*

c. *What has happened to the amplitude and period as compared to the results in Figure 3?*

EXERCISE 3 – A Change of the Period, $y = Sin(t/2)$

a. *Modify the program in Figure 2 to plot $y = Sin(t/2)$.*

b. *Sketch the output you expect. Then run the program and compare with your sketch.*

c. *What has happened to the amplitude and period compared to the results in Figure 3?*

EXERCISE 4 – Discovery

Use the computer to find out what happens when a constant angle is added to the argument of a sine function as $y = Sin(\omega t + \phi)$. Specifically, investigate $y = Sin\left(t + \frac{\pi}{4}\right)$. Draw a sketch of the expected results and compare with your computer output. Try to analyze the results in terms of the unit circle in Figure 1.

ADVANCED EXERCISES

EXERCISE 5 – Sum of Two Sinusoids with Same Period

Suppose we add two sinusoids together as $y = Cos(t) + Sin(t)$? [Note that $Cos(\omega t)$ is also known as a sinusoid.] Will the period change? What about the amplitude? Explain your results in terms of the unit circle in Figure 1.

EXERCISE 6 – A New Function

Given $y = 3\ Sin(4x)$

a. What is the amplitude?

b. What is the wavelength?

c. What is the wave number?

EXERCISE 7 – Discovery

Sketch the output you would expect from $y = Sin^2(x) + Cos^2(x)$. Verify your results on the computer. Justify these results in terms of the unit circle in Figure 1.

EXERCISE 8 – Sum of Two Sinusoids with Different Periods

a. Plot the shape of $y = Cos(t) + Sin(2t)$.

b. Has the amplitude of the sum changed from that of either $Cos(t)$ or $Sin(2t)$?

c. Has the period changed?

d. Is the sum periodic?

STANDING WAVES

The first type of wave we will study is known as a standing wave. The study of standing waves has a widespread application. We already have most of the background we need to study them. The additional vocabulary and concepts we will need will be added in the discussion.

We already know how y = Cos(ωt) behaves (see the results of Exercise 1). As time changes, the value of y traces out the cosine wave, repeating it every t seconds where t is the period. The amplitude of y is 1 because the greatest value taken on by y as t changes is ±1.

We have identified ω as the *angular velocity* with which a point moves around a unit circle, in order to describe the behavior of the sine and cosine functions. However, it is more accurate to define ω as the *angular frequency*; that is, the frequency of the sinusoid measured in radians per second. Since there are 2π radians per cycle we can also define the *cyclical frequency*, f, which is measured in cycles per second. The relationship between ω and f is then

$$f = \frac{\omega}{2\pi} . \qquad (9)$$

Thus, Sin(ωt) and Sin(2πft) are equivalent.

We will also need to be more careful in describing y, for example as in y = Sin(ωt). We must indicate that y is a function of t by writing y(t) = Sin(ωt). [y(t) is read "y of t."] We need to start identifying y explicitly because the function y for the waves we will study depends upon *both* x and t. We will indicate this by writing y(x,t) (which is read as "y of x and t"). This notation does *not* mean the product of y, x, and t, but is merely a convenient way to show that y depends on both x and t.

All standing waves can be expressed in the form

$$y(x,t) = A(x)\mathrm{Cos}(\omega t + \phi) . \qquad (10)$$

This states that the displacement, y(x,t), of a point on the wave is the product of an amplitude, A(x), which is itself a function of x, and a cosine function, Cos(ωt + ϕ). We will look at each of the terms on the right side of (10) separately.

First, the amplitude A(x) gives the *shape* of the standing wave. A(x) is an arbitrary function of x which yields the desired shape when plotted. For instance, consider a string stretched between two points (0,0) and (L,0). Suppose that we want to deform this string into the *shape* equivalent to a parabola passing through the points (0,0), (L/2,L/4), and (L,0). It can be shown that the equation of this *shape curve* is

$$A(x) = \frac{-x^2}{L} + x . \qquad (11)$$

Note that we could have chosen *any* A(x): all it specifies is the shape.

Now, let us look at a particular point, say x = a, and see what happens as time changes. The amplitude at x = a would be given by A(a), which would be the amplitude of the motion described by (10). The equation which correctly describes the motion at x = a is then

$$y(a,t) = A(a)\text{Cos}(\omega t + \phi)\ . \tag{12}$$

Suppose that $\phi = 0$. In this case, we know how the function behaves. When t = 0, $y(a,0) = A(a)\text{Cos}(0) = A(a)$, and likewise $y\left(a,\frac{\pi}{2\omega}\right) = 0$, $y\left(a,\frac{\pi}{\omega}\right) = -A(a)$, and so on. Thus, the point on the standing wave at x = a moves up and down according to $\text{Cos}(\omega t)$ as time changes. If $\phi \neq 0$, the whole movement is merely shifted in time (see the results of Exercise 4).

Any point on the wave can be analyzed in this way. Thus we see that the *whole wave* changes shape as t changes. Figure 4 lists a program which investigates

$$y(x,t) = \left(-\frac{x^2}{L} + x\right)\text{Cos}(\pi t)\ . \tag{13}$$

In Line 130, t can be input to generate any argument of the cosine function desired.

```
LIST
100  REM STANDING WAVE
110  LET S=10
120  LET L=10
130  INPUT F
140  LET P=3.14159
150  DEF FNA(X)=(-X↑2/L+X)*COS(F*P)
160  FOR I=1 TO 60
170  PRINT TAB(I);"-";
180  NEXT I
190  PRINT
200  FOR X=0 TO 10.0001 STEP .5
210  LET Y=INT(S*FNA(X)+30.5)
220  IF Y>30 THEN 260
230  IF Y<30 THEN 280
240  PRINT TAB(30);"*"
250  GOTO 290
260  PRINT TAB(30);"I"; TAB(Y);"*"
270  GOTO 290
280  PRINT TAB(Y);"*"; TAB(30);"I"
290  NEXT X
999  END

READY
```

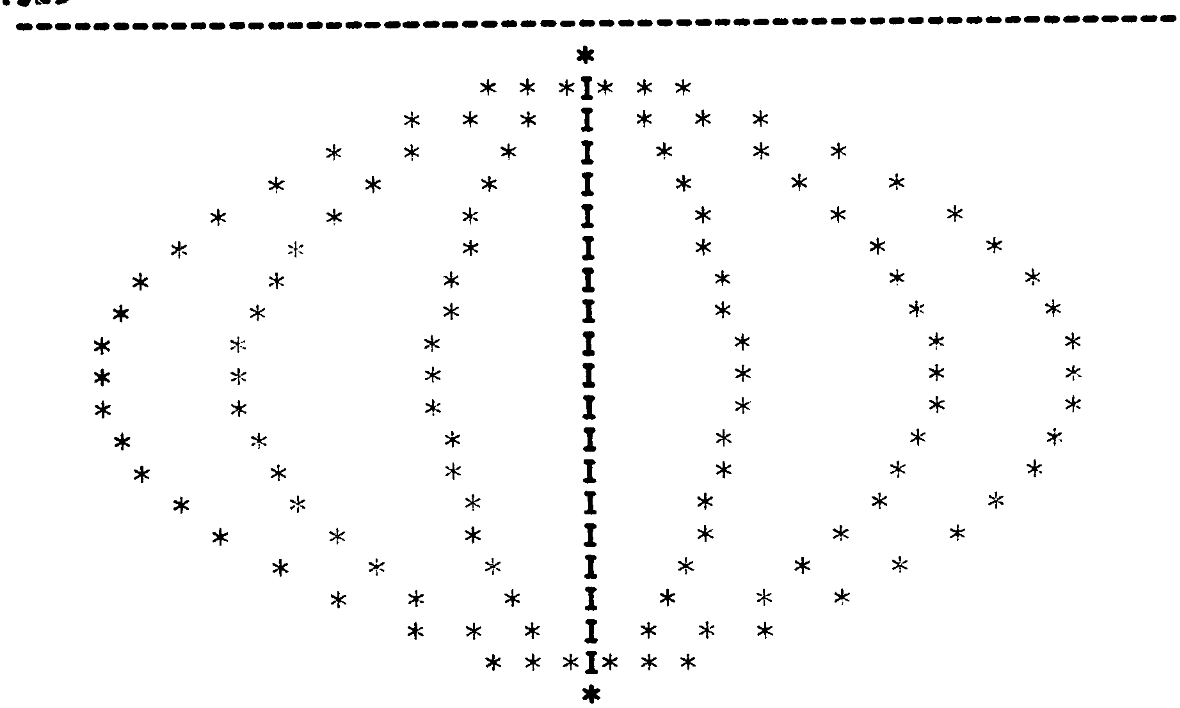
Figure 4 – Program for Standing Wave

Our strategy will be to fix a value of t, then generate the shape of the curve and plot it. Figure 5 was generated by running this program for several values of t, thus show-

ing how the wave moves as t changes. A simple trick was used to superimpose each run over the previous results. Merely roll the paper in the teletype backwards so that you type RUN directly over the same command for the previous computation. In this fashion several curves can be generated on the same plot.

The results clearly justify our prediction that the shape of the standing wave changes as t changes. As t increases, the whole wave flattens out and reverses itself on the negative side of the x axis. If we continued plotting curves for increasing values of t, the wave would return to its original position and shape. Thus we can define the *period* with which the wave goes through one complete cycle (or oscillation) which in turn can be used to give the *frequency* of the vibration in cycles per second.

```
RUN
?[illegible]
```



```
READY
```

Figure 5 – Computer Plot of Standing Wave

Now think of a standing wave on a string of length L where the *shape* of the wave is a sinusoid. We will require that the ends of the string remain fixed. This wave has an amplitude given by $A(x) = A \,\text{Sin}(kx)$. However, if the ends of the string are to remain fixed, only certain values of k are permissible [remember that k is related to the wavelength of the sinusoid by (8)]. Now, by looking at a picture of the sine wave we can see that we could have, for example, $L = \frac{\lambda}{2}$, or $L = \lambda$. However, there are many other possibilities which would also be acceptable. In general we can have

$$L = \frac{n}{2}\lambda, \text{ where } n = 1,2,3,\cdots \tag{14}$$

From (14) we can solve for λ, which in turn can be used to solve for k in (8). The result is

$$k = \frac{n\pi}{L}. \qquad (15)$$

The full expression for the standing wave is obtained at this point by substituting (15) into A(x) which in turn is substituted into (10) to give

$$y(x,t) = A\,\mathrm{Sin}\left(\frac{n\pi}{L}x\right)\mathrm{Cos}(2\pi ft + \phi)\text{, where } n = 1,2,3,\cdots \qquad (16)$$

Before proceeding to exercises, it is important to make sure you understand all parts of the expression. The displacement, which in the case of a vibrating string is the distance a point on the string is away from the straight line or equilibrium position, is y(x,t) and depends on both x and t. The maximum displacement possible is A. The shape of the standing wave is given by the product of A and the sine term. Finally, the rate at which the wave vibrates is given by the cosine term. In the argument of the cosine term, f is the cyclical frequency measured in cycles per second, and ϕ is some additive constant which is called a *phase angle*. ϕ determines where the wave is in the vibration cycle when t = 0. ϕ can be set equal to zero without serious loss of generality.

EXERCISE 9 – Standing Wave Computation

Adapt the program in Figure 4 to investigate

$$y(x,t) = 2\mathrm{Sin}\left(\frac{\pi}{10}x\right)\mathrm{Cos}(t)$$

assuming that L = 10. Plot curves for t = 0, $\pi/4$, $\pi/2$, $3\pi/4$, and π.

a. What is the maximum amplitude of the wave?

b. What is the wavelength of the standing wave?

c. What is the cyclical frequency of the standing wave?

d. What important fact can you deduce from the curve when t = $\pi/2$?

EXERCISE 10 – Standing Wave Computation

Modify the program in Exercise 9 to investigate

$$y(x,t) = 2\mathrm{Sin}\left(\frac{\pi}{5}x\right)\mathrm{Cos}(t)$$

assuming that L = 10.

a. Before running the program, determine the maximum amplitude, wavelength, and cyclical frequency. Draw a sketch of the anticipated results.

b. Run the program to verify your predictions.

EXERCISE 11 – Standing Wave Computation

Repeat Exercise 10 for the following:

$$y(x,t) = 3Sin\left(\frac{3\pi}{10}x\right)Cos(2\pi t)\,.$$

EXERCISE 12 – Nodes

A point on a standing wave that has zero displacement for all values of t is called a node. Find the nodes on the standing wave

$$y(x,t) = 10Sin\left(\frac{4\pi}{10}x\right)Cos(50\pi t)$$

assuming that L = 10. Use the computer to verify your answer.

ADVANCED EXERCISES

EXERCISE 13 – Superposition of Standing Waves (Different Cyclical Frequencies)

Use the computer to find out what happens when two standing waves of the same amplitude and wavelength but different cyclical frequencies are added together. Is the result a standing wave?

EXERCISE 14 – Superposition of Standing Waves (Different Wavelengths)

Use the computer to find out what happens when two standing waves of the same amplitude and frequency but different wavelengths are added together. Is the result a standing wave?

EXERCISE 15 – Superposition of Standing Waves (Different Wavelengths and Frequencies)

Use the computer to find out what happens when two standing waves of the same amplitude but different wavelengths and frequencies are added together. Is the result a standing wave?

EXERCISE 16 – Trigonometric Superposition

If you have completed a course in trigonometry, try to solve Exercises 13, 14, and 15 analytically.

TRAVELING WAVES

Now we can look at a fundamentally different type of wave, the *traveling wave*. The standing wave was generated by a product of a function of x times a function of t. We could therefore consider the spatial or x part of the wave and the time part of the wave separately. For traveling waves, the time and spatial properties always occur together. An example of a traveling wave is

$$y(x,t) = \mathrm{Cos}(t - x)\ . \tag{17}$$

The program listed in Figure 6 computes the value of y(x,t) over a range of values of x for a range of values of t. The result is shown in Figure 7. The difference between successive values of x is $\pi/2$, which is also the spacing in time. Each of the lines of data represents displacements for a fixed value of t. Note that as time increases in successive lines the entire displacement pattern shifts one step to the right, hence the name *traveling waves.*

The more general representation of a traveling wave similar to that in (17) is

$$y(x,t) = A\ \mathrm{Cos}(\omega t - kx)\ . \tag{18}$$

If we hold t constant in (18) and vary x, the result is a sinusoid in space. If we hold x constant and vary t, the result is a sinusoid in time. ω and k are still defined as they were for standing waves.

```
LIST
100  REM TRAVELING WAVE
110  LET P=3.14159
120  PRINT
130  PRINT "X1","X2","X3","X4","X5"
140  PRINT
150  FOR T=0 TO 2*P STEP P/2
160  PRINT
170  FOR X=0 TO 2*P STEP P/2
180  LET Y=COS(T-X)
190  PRINT Y,
200  NEXT X
210  NEXT T
999  END

READY
```

Figure 6 – Program for Traveling Wave

```
RUN
X1              X2              X3              X4              X5


 1.             1.19209E-06    -1.             -3.74507E-06     1
 1.12352E-06    1.              1.19209E-06    -1.             -3.74507E-06
-1.             1.12352E-06     1.              1.19209E-06    -1.
-3.74507E-06   -1.              1.12352E-06     1.              1.19209E-06
 1             -3.74507E-06    -1.              1.12352E-06     1.
READY
```

Figure 7 – Output from Traveling Wave Program

Now, suppose we look at a peak on the wave, i.e., when $\cos(\omega t - kx) = 1$. There will be a peak any time $\omega t - kx = 0$. Consequently, if the peak is at x_1 at time t_1, and at x_2 at time t_2 we can write $\omega t_1 - kx_1 = 0$, and $\omega t_2 - kx_2 = 0$. These equations can be put into the form

$$\frac{x_2 - x_1}{t_2 - t_1} = \frac{\omega}{k}. \tag{19}$$

Therefore $(x_2 - x_1)/(t_2 - t_1)$ is the velocity of the peak of the wave as it moves to the right. Since the peak is determined by zero phase difference between ωt and kx ($\omega t - kx$ must equal zero) the velocity is known as the *phase velocity*.

$$v_\phi = \frac{\omega}{k} \tag{20}$$

Equations (18), (19) and (20) describe the behavior of a traveling sinusoid. It turns out that a more fundamental description is

$$y(x,t) = A\, f(\omega t - kx)\ , \tag{21}$$

where $f(\omega t - kx)$ refers to *any* function at all as long as the argument of the function is $\omega t - kx$. The traveling waves are consequently not limited to sinusoid shapes but can have arbitrary shapes determined by the function f.

EXERCISE 17 – Traveling Wave Computation

What is the phase velocity of the following traveling wave?

$$y(x,t) = 2\mathrm{Cos}(2t - \pi x/10)$$

Adapt the program in Figure 4 to verify the answer.

EXERCISE 18 – Traveling Wave

What is the phase velocity of the following traveling wave?

$$y(x,t) = \mathrm{Cos}(t + \pi x/5)$$

Adapt the program in Figure 4 to verify your answer.

ADVANCED EXERCISES

EXERCISE 19 – Superposition of Traveling Waves

What is the result of adding two traveling waves which are traveling in opposite directions? Assume that ω and k are the same for both waves. Use the computer to verify your answer.

EXERCISE 20 – Traveling Gaussian Wave

Use the computer to investigate the characteristics of

$$y(x,t) = e^{-\left(\frac{t - x}{2}\right)^2}.$$

Note: e is a mathematical constant. Your instructor can show you how to generate functions of e on the computer.

SUPERPOSITION OF SINUSOIDS

In the previous sections, we have looked briefly at the *superposition* or *adding together* of waves. In this section we will examine the subject of superposition of sinusoids (sine and cosine functions) in detail.

Suppose that we have a superposition given by

$$y(t) = y_1(t) + y_2(t)\,, \tag{22}$$

where $y_1 = A\ \mathrm{Cos}(t + \phi)$, and $y_2 = A\ \mathrm{Cos}(t)$. The superposition is of two sinusoids with identical amplitude and frequency. However, one of the sinusoids has an arbitrary phase constant ϕ. We have already looked at this question briefly (see Exercise 4). The central issue is the phase constant ϕ. For certain values of ϕ, $y(t)_{max} = 2A$. This condition is called *constructive interference*. For certain other values of ϕ $y(t)_{max} = 0$. This condition is called *destructive interference*. These ideas will be explored further in the exercises.

Another kind of superposition is expressed as

$$y(x) = y_1(x) + y_2(x) + y_3(x) + \cdots \tag{23}$$

where $y_1(x) = A_1\,\mathrm{Sin}(k_1 x)$, $y_2(x) = A_2\,\mathrm{Sin}(k_2 x)$, and so on. In the exercises you will discover that through this kind of superposition it is possible to build shapes that show very little relationship to the sinusoids in the superposition. This is a very powerful procedure which finds widespread application in physics. The superposition in (23) is an example of a *Fourier Series*. We will not try to derive the series used in the exercises, but will study them to become familiar with the process described above.

EXERCISE 21 – Interference of Sine Waves

Write a computer program to plot $y(t) = \mathrm{Sin}(2\pi t + \phi) + \mathrm{Sin}(2\pi t)$ for $0 < t < 1$. Run the program for $\phi = 0, \pi/4, \pi/2, 3\pi/4$, and π. Identify the plots where constructive and destructive interference take place.

EXERCISE 22 – Interference of Cosine Waves

Repeat Exercise 21 for $y(t) = 2\mathrm{Cos}(2\pi t + \phi) + \mathrm{Cos}(2\pi t)$.

EXERCISE 23 – Interference of Sine and Cosine Waves

Repeat Exercise 21 for $y(t) = \mathrm{Sin}(2\pi t + \phi) + \mathrm{Cos}(2\pi t)$.

EXERCISE 24 – Intensity

The intensity of a superposition is proportional to the square of the amplitude. Assume that if $I(t)$ stands for the intensity that $I(t) = [y(t)]^2$. Write a

program to plot the intensity of the superposition in Exercise 21 for each of the values of ϕ specified.

ADVANCED EXERCISES

EXERCISE 25 – Discovery

Write a program to plot

$$y(x) = \frac{4}{1\pi}Sin\left[\frac{1\pi x}{5}\right] + \frac{4}{3\pi}Sin\left[\frac{3\pi x}{5}\right] + \frac{4}{5\pi}Sin\left[\frac{5\pi x}{5}\right] + \cdots$$

Study the series until you are sure you understand the pattern. Then arrange your program to plot a given number of terms in the series. Make several plots, increasing the number of terms in each plot until you can identify the shape that is being generated. Plot over the range $0<x<10$.

EXERCISE 26 – An Unknown Wave Form

Repeat Exercise 25 but use the series below.

$$y(x) = \frac{2(-1)^2}{1\pi}\,Sin\left[\frac{1\pi x}{5}\right] + \frac{2(-1)^3}{2\pi}\,Sin\left[\frac{2\pi x}{5}\right] + \frac{2(-1)^4}{3\pi}\,Sin\left[\frac{3\pi x}{5}\right] + \cdots$$